Warm-up!

$24 \rightarrow \boxed{f} \rightarrow 7 \qquad 7 \rightarrow \boxed{f^{-1}} \rightarrow 24$

Periodic Graphs Day 10:
(2.8) Inverse Functions

Homework: Inverse Functions Practice

Upcoming:
Desmos Quiz on Day 12
Test on Day 13

These are review if you remember…

A) Let $f(x) = -- 1$. Find $f^{-1}(7)$. $= 24$

$7 = \frac{x}{3} - 1$

$8 = \frac{x}{3} \longrightarrow x = 24$

B) Let 4. Find). $= \sqrt[3]{x+4}$

$x = y^3 - 4$

$x + 4 = y^3$

$y = \sqrt[3]{x+4}$

C) Let -. Find all asymptotes of).

h has asymptotes

$y = 2$

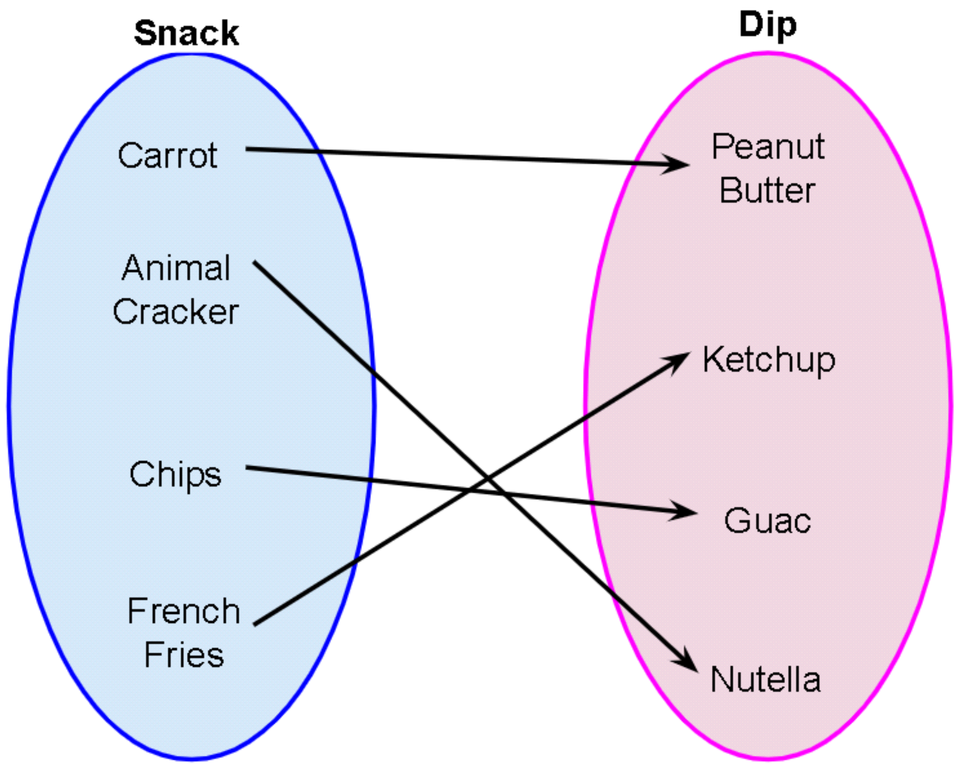
$x = -5$

h^{-1} has asymptotes

$x = 2$

$y = -5$

One way to think about it: Inverse functions "undo" each other.



Consider the function $h(t)$ that inputs how many days it has been since the start of the school year and outputs how many million people have seen the movie K Pop Demon Hunter. For instance $h(5) = 203$ means that by five days after the school year began, 203 million people had already watched K Pop Demon Hunters.

What does $h^{-1}(250) = 20$ represent?

days since school began $\to \boxed{h} \to$ # viewers (in millions)

$250 \to \boxed{h^{-1}} \to 20$

$20 \to \boxed{h} \to 250$

Consider the function f that inputs the number of people who stop at a gas station and outputs a prediciton for how many bags of chips the gas station will sell. And consider g that inputs the price of gas and outputs how many people will stop at the gas station.

$\# \text{ people} \longrightarrow \boxed{f} \longrightarrow \# \text{ bags of chips}$

$\text{price} \longrightarrow \boxed{g} \longrightarrow \# \text{ people}$

What does f^{-1} input and output?

$\# \text{ bag of chips} \longrightarrow \boxed{f^{-1}} \longrightarrow \# \text{ people}$

What does g^{-1} input and output?

$\# \text{ people} \longrightarrow \boxed{g^{-1}} \longrightarrow \text{price}$

What does $\left(g^{-1} \circ f^{-1}\right)(x)$ input and output?

$g^{-1}\left(f^{-1}(x)\right)$

$\# \text{ bags} \longrightarrow \boxed{f^{-1}} \xrightarrow{\# \text{ people}} \boxed{g^{-1}} \longrightarrow \text{price}$

Let $f(x) = 3x + 10$

Evaluate $f(4)$ $= 3(4)+10$

$12+10$

22

$4 \to \boxed{f} \to 22$

Evaluate $f^{-1}(4)$ $= -2$

$4 = 3x+10$

$-6 = 3x$

$-2 = x$

$4 \to \boxed{f^{-1}} \to -2$

$-2 \to \boxed{f} \to 4$

Let $g(x)$ be an invertible function (AKA "one-to-one") with values given in the table below.

x	$g(x)$
-2	1
0	2
1	3
3	5
5	7
6	10

A) Find $g^{-1}(3) = 1$

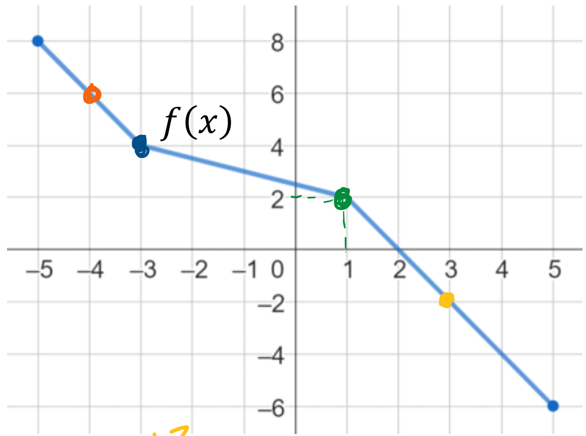
$g(1) = 3$

B) Find $g^{-1}(1) = -2$

$g(-2) = 1$

C) Find $g^{-1}\left(g^{-1}(7)\right)$

$g^{-1}(5) = 3$



$-2 = \frac{12}{x}$

Let $g(x) = \frac{12}{x}$ for $x \neq 0$

$2 = \frac{12}{x}$

A) $f^{-1}(2) = 1$

B) $f^{-1}(4) = -3$

C) $(f^{-1} \circ g^{-1})(2) =$

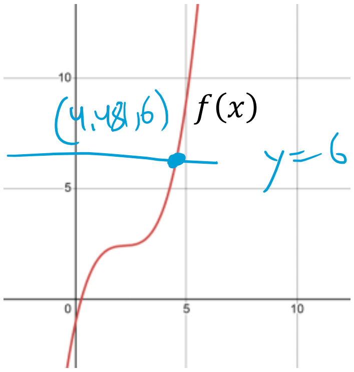
$f^{-1}\left(g^{-1}(2)\right) = f^{-1}(6) = -4$

D) $(g^{-1} \circ f)(3)$

$g^{-1}\left(f(3)\right) = g^{-1}(-2) = -6$

Let's try doing that with Desmos

Use Desmos to graph $f(x) = 0.3x^3 - 2x^2 + 4.5x - 1$.
The graph should look something like the graph shown below



A) $f^{-1}(6)$ $= 4.481$

B) $f^{-1}(1)$ $= 0.582$

C) $(f^{-1} \circ g^{-1})(2)$

$f^{-1}(g^{-1}(2)) = f^{-1}(3) = 3.422$

D) $(g^{-1} \circ f)(3)$

$g^{-1}(f(3)) = g^{-1}(2.6) = 5.76$

Let $g(x) = \sqrt{x+1}$ for $x > 0$

$2 = \sqrt{x+1}$

$2.6 = \sqrt{x+1}$

The x y switcheroo

A) If the graph of $y = f(x)$ contains the point $(2,5)$, what do we know about the graph of $f^{-1}(x)$?

$f^{-1}(x)$ contains $(5,2)$

B) If the graph of $y = f(x)$ has domain $(-4, \infty)$, what do we know about the graph of $f^{-1}(x)$?

$f^{-1}(x)$ has range $(-4, \infty)$

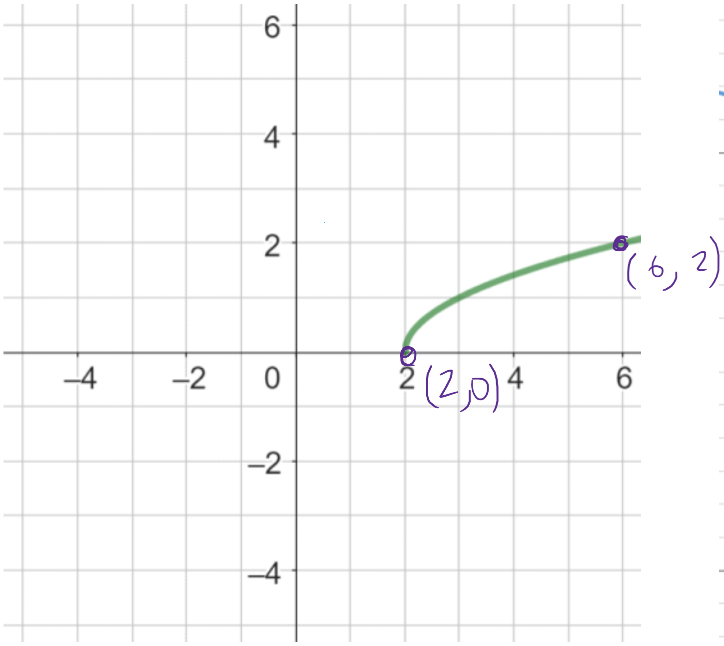
C) If the graph of $y = f(x)$ has range $[-10,4)$, what do we know about the graph of $f^{-1}(x)$?

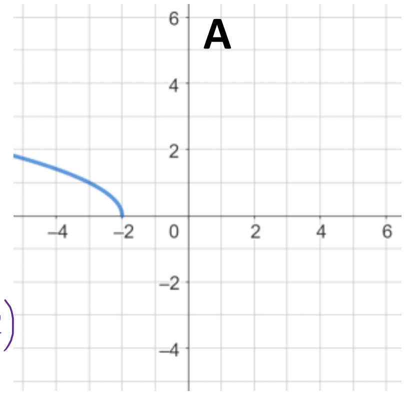
$f^{-1}(x)$ has domain $[-10, 4)$

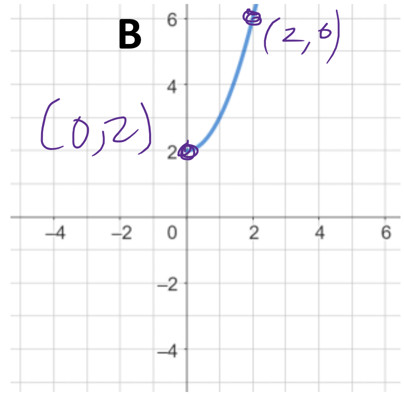
D) If the graph of $y = f(x)$ has asymptote $x = 4$, what do we know about the graph of $f^{-1}(x)$?

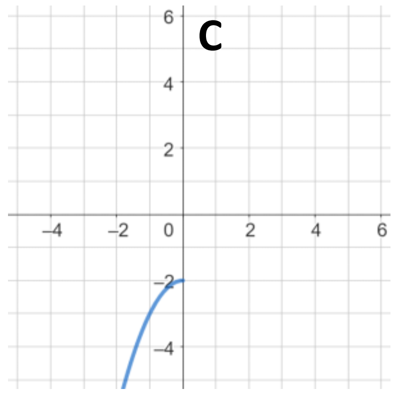
$f^{-1}(x)$ has asymptote $y=4$

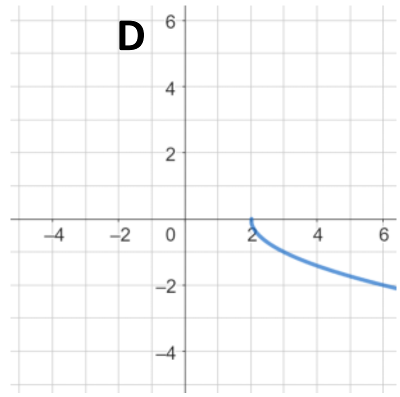
On the left is the graph of $f(x)$. Of the four smaller graphs to the right, determine which is the graph of $f^{-1}(x)$.

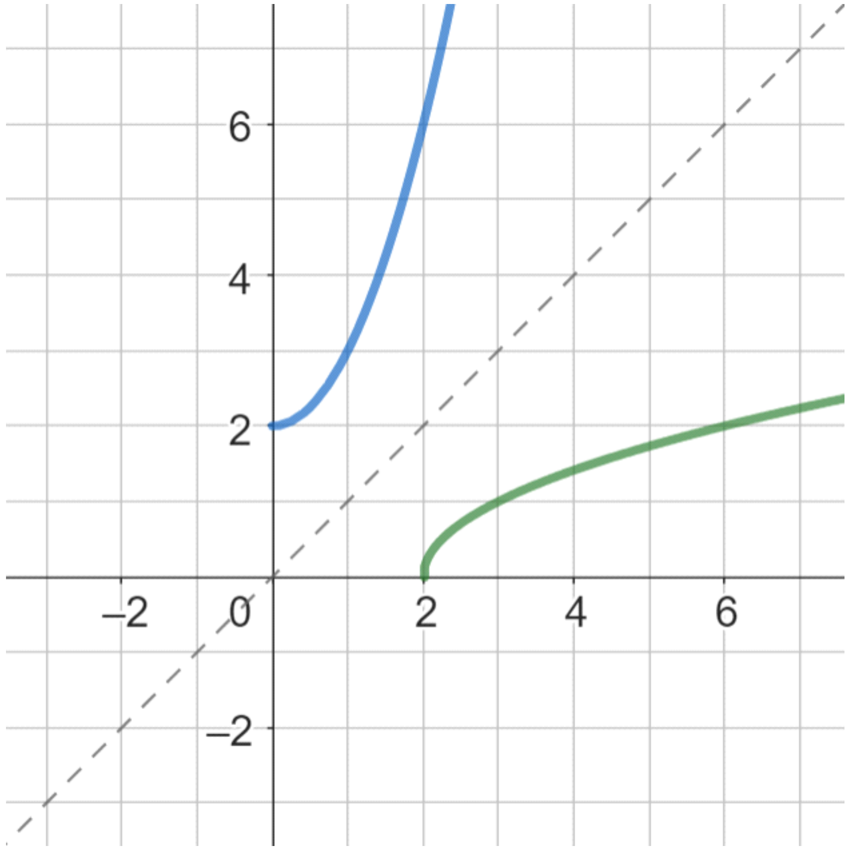






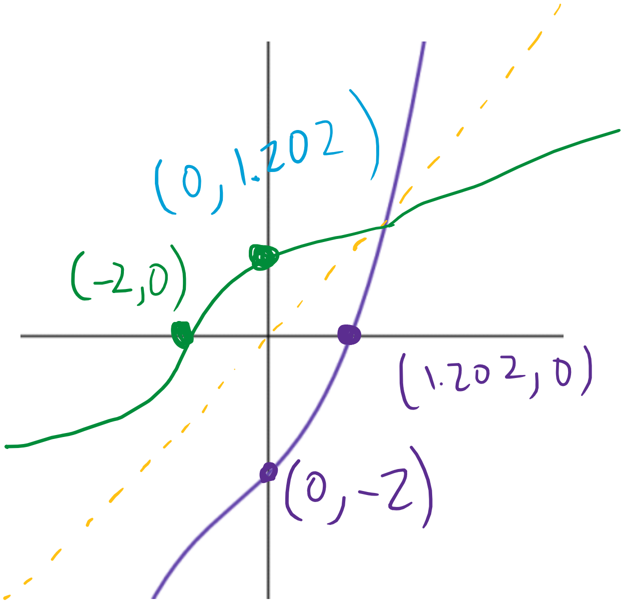





6
4
2
–2
–2
0
2
4
6

Below is the graph of $y = f(x) = 0.21x^3 + \frac{3}{10}x^2 + x - 2$

Use Desmos to obtain a graph of $f(x)$. It should look like the graph shown below.



Roughly sketch what you believe the graph of $f^{-1}(x)$ would look like.

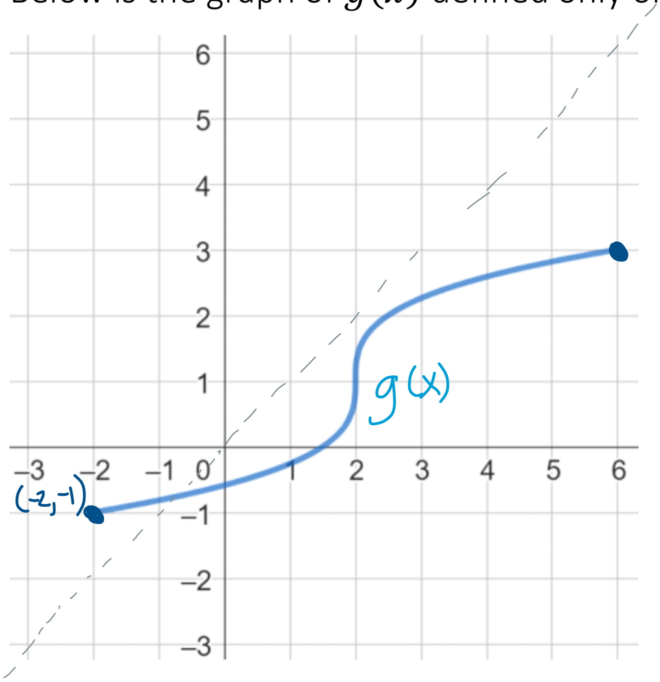
Using Desmos, find the y-intercept of $f^{-1}(x)$

(0, 1.202)

Then, find the x-intercept of $f^{-1}(x)$

(-2, 0)

Below is the graph of $g(x)$ defined only on the interval $-2 \leq x \leq 6$



A) Roughly sketch g^{-1} on the graph of g

B) What is the absolute minimum value of g^{-1}?

C) Over what interval is g^{-1} increasing?

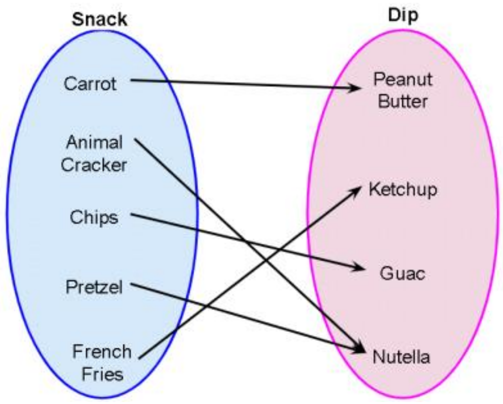
D) Over what interval is g^{-1} concave down?

So far we have learned/remembered

- When asked $f^{-1}(c)$, c is the output (y-value) of f and we need to find the corresponding input (x-value) of f.
- Comparing graphs of f and f^{-1}, the role of x and y swap
  - The point (a, b) becomes the point (b, a)
  - Domains become ranges, ranges become domains
  - Horizontal asymptotes $y = \#$ become vertical asymptotes $x = \#$
- Comparing graphs of f and f^{-1}, they look like reflections of each other over the line $y = x$.

"One To One"

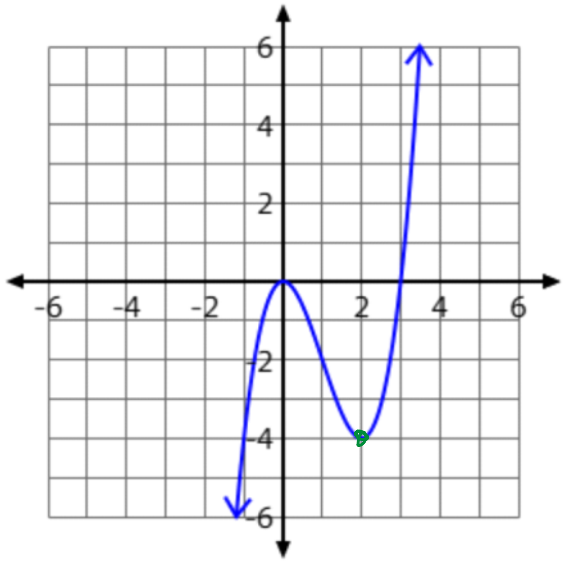
Not all functions have an inverse function



When a function does have an inverse function, we call it "one-to-one"

invertible

Consider the graph of $f(x)$ shown below



A) Determine if f has an inverse function

No

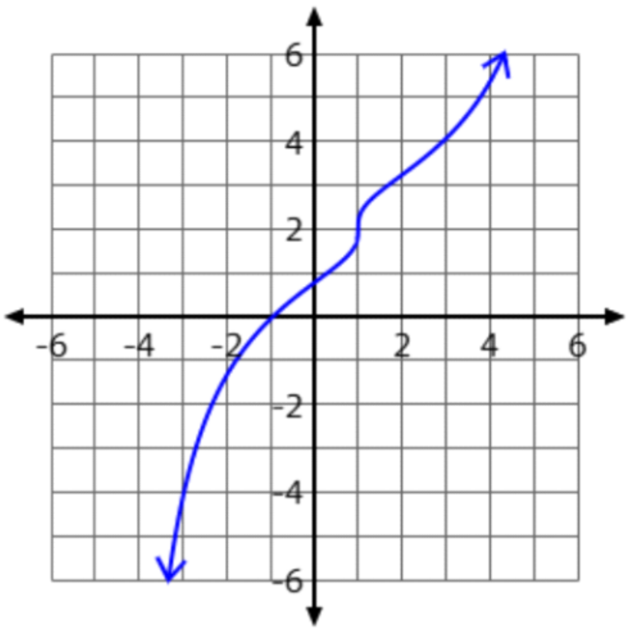
B) Give a reason based on the graph of f. Refer to specific values from the graph.

$f(-1) = 4$ and $f(2) = 4$

$y = 4$ happens at two different x-values

C) If f does not have an inverse, is there a way to restrict the domain so that f does have an inverse?

Consider the graph of $g(x)$ shown below



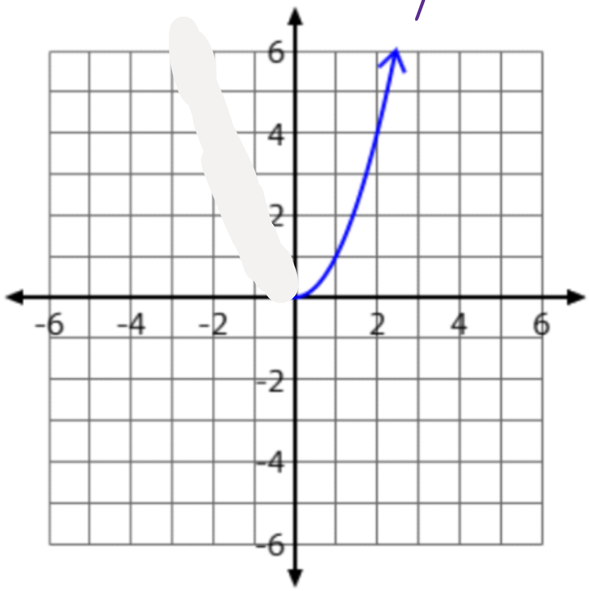
A) Determine if g has an inverse function

B) Give a reason based on the graph of g. Refer to specific values from the graph.

C) If g does not have an inverse, is there a way to restrict the domain so that g does have an inverse?

Consider the graph of $h(x)$ shown below

$y = x^2, \; x \geq 0$ $\quad$ $\sqrt{x}$ $\quad$ Range: $[0, \infty)$



A) Determine if h has an inverse function

B) Give a reason based on the graph of h. Refer to specific values from the graph.

C) If h does not have an inverse, is there a way to restrict the domain so that h does have an inverse?

Proving Two Functions Are Inverses

You must show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$
(note: beyond this being a technique to show the functions are inverses, this is the definition of inverse functions)

Example:
Show $f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$ are inverses.

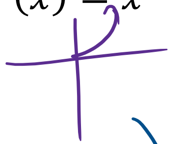
$$f(g(x)) = f\left(\frac{x-9}{4}\right) = 4\left(\frac{x-9}{4}\right)+9 = (x-9)+9$$
$$= x$$

$$g(f(x)) = g(4x+9) = \frac{(4x+9)-9}{4} = \frac{4x}{4} = x$$

Are $f(x) = \frac{3}{x} + 2$ and $g(x) = \frac{2}{x} - 3$ inverses? Prove it.

$x \geq 0$

Are $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses? Prove it.



$$f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

$$g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$$

Finding Inverse Functions

Step 1: Swap x and y
Step 2: Isolate y
Step 3: Profit

Example: Let $f(x) = \frac{2x-1}{3}$. Find an expression for $f^{-1}(x)$.

$y = \frac{2x-1}{3}$ ← original

$x = \frac{2y-1}{3}$ ← inverse

$3x = 2y - 1$

$3x + 1 = 2y$

$\frac{3x+1}{2} = y$

$f^{-1}(x) = \frac{3x+1}{2}$

Find the inverse, if possible

B) $g(x) = \sqrt[3]{x + 8}$

$g^{-1}(x) = x^3 - 8$

$x = \sqrt[3]{y+8}$

$x^3 = y + 8$

$x^3 - 8 = y$

Find the inverse, if possible

C) $t(x) = \frac{2}{x+1}$

Find the inverse, if possible

D) $h(x) = x^2 - 4x + 1$

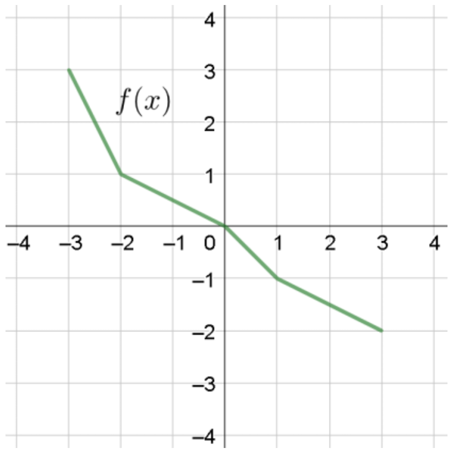
Find the inverse, if possible

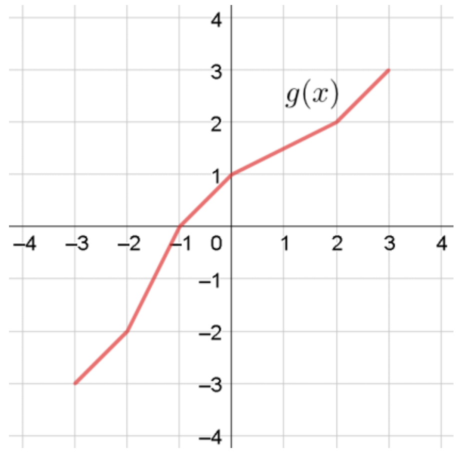
E) $k(x) = \frac{x}{x-4}$

More that we've learned/remembered today

- A function has an inverse if it passes both the vertical line test (VLT) and horizontal line test (HLT). The term for such a function is "one-to-one."
- To verify two functions are inverses of each other, show $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$
- Sometimes we have to restrict the domain of a function to make it invertible (AKA "to make it one-to-one")
- To find the inverse function equation, swap x and y, then solve for y.

Find the indicated values if possible





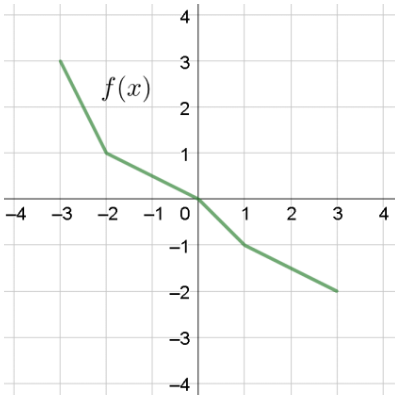
A) $f^{-1}(3)$

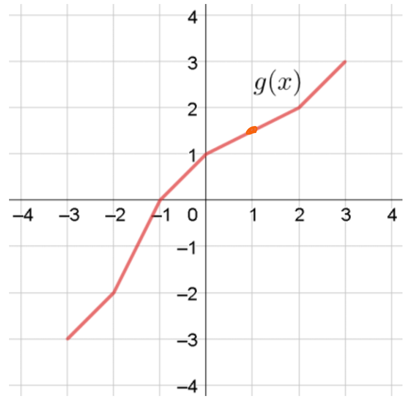
B) $g^{-1}(0)$

C) $g^{-1}(1)$

D) $f^{-1}(0)$

Find the indicated values if possible





E) $(f \circ g)(-1)$

$f(g(-1))$

$f(0)$

0

F) $(g \circ f)(-2)$

$g(f(-2))$

$g(1)$

1.5

G) $(f \circ g^{-1})(1)$

$f(g^{-1}(1))$

$f(0)$

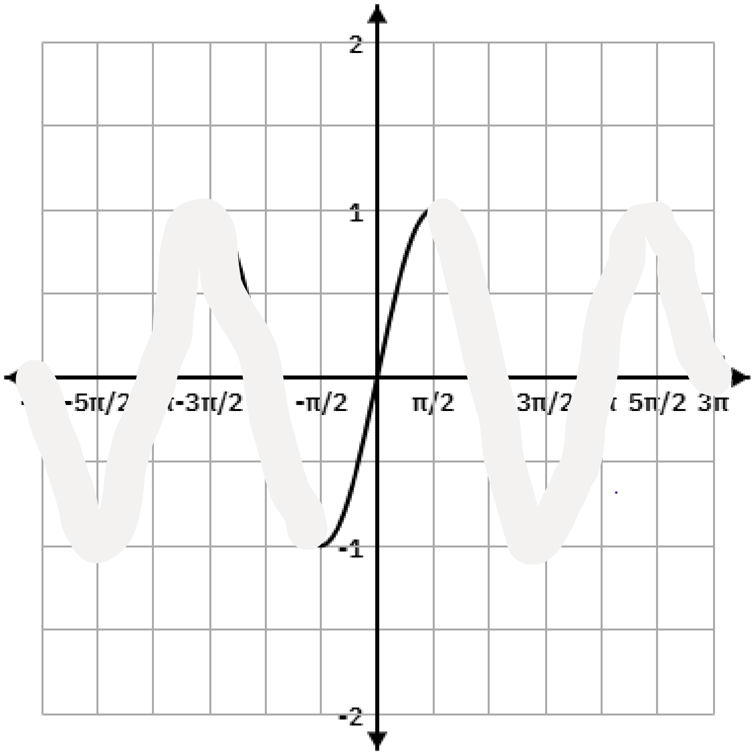
0

H) $(g \circ f^{-1})(-1)$

$g(f^{-1}(-1))$

$g(1)$

1.5



$f(x) = \sin x$

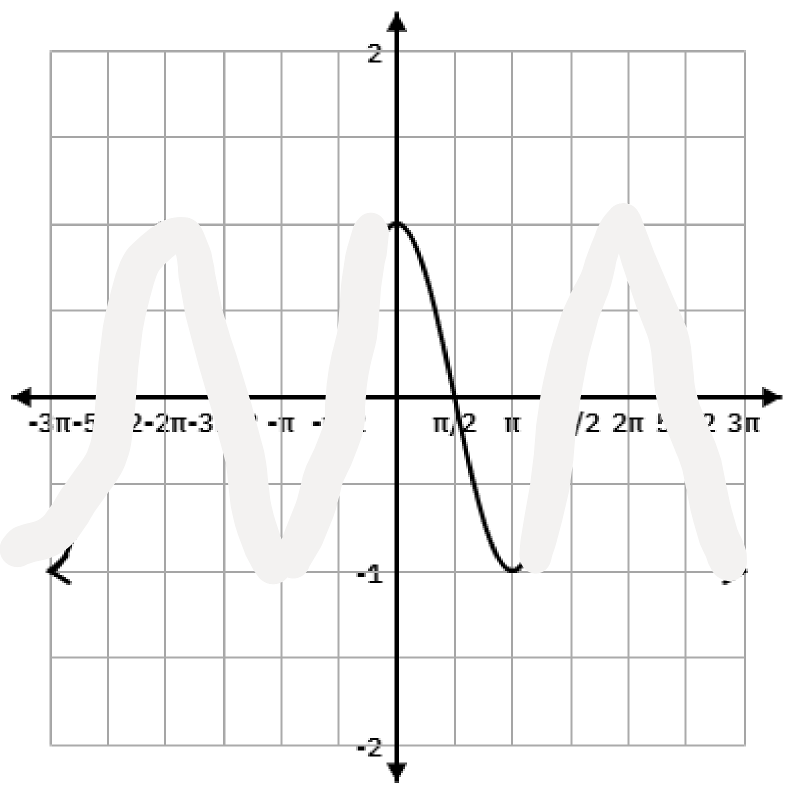
restrict domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

range $[-1, 1]$

$f^{-1}(x) = \arcsin(x)$

domain: $[-1, 1]$

range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

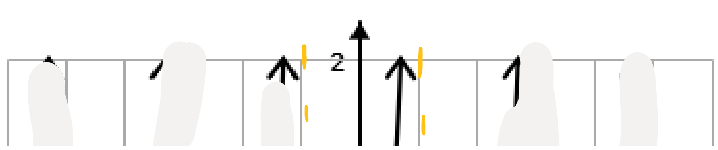


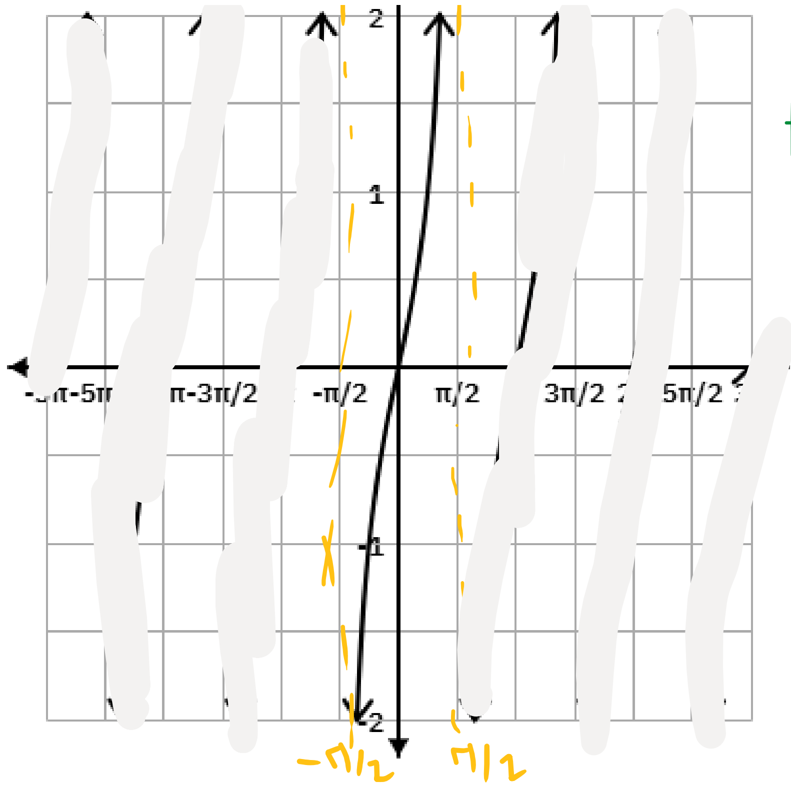
$f(x) = \cos x$

domain: $[0, \pi]$

$f^{-1}(x) = \arccos(x)$

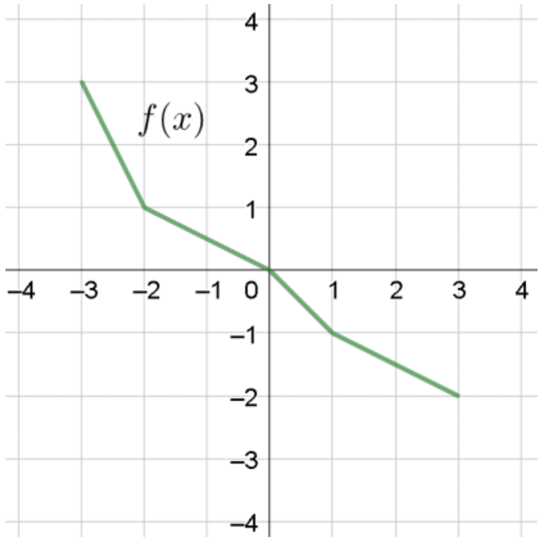
range: $[0, \pi]$

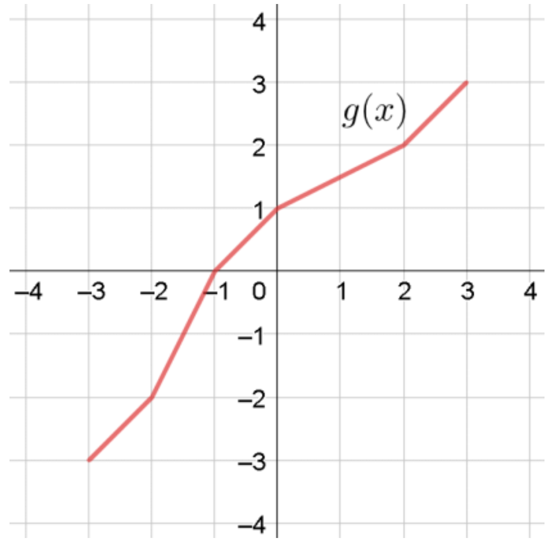




$f(x) = \tan x$

Find the indicated values if possible





I) $(f \circ f^{-1})(2)$

J) $(g^{-1} \circ f^{-1})(2)$

$f(x) = x + 4$ $\qquad$ $g(x) = \sqrt{x} - 5$ $\qquad$ $h(x) = x^2 + x, x \geq -\frac{1}{2}$

Find the indicated value.

A) $f^{-1}(0)$

B) $g^{-1}(2)$

C) $h^{-1}(2)$